

Effects of thermal and solutal dispersions on free convective flow of Newtonian fluid over a cone in a non-Darcy porous medium

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-----ABSTRACT-----

The Newtonian fluid flow due to free convection over an isothermal cone with dispersions of thermal and solutal transport situated in a non-Darcy porous medium, is considered in this paper. Numerical solution of the non-dimensional representation of governing equations are provided using the shooting method in conjunction with the Runge-Kutta technique of 4th order and also provided the comparison table to show the accuracy of present investigation. The variation of physical quantities is discussed in non-Darcian porous medium for dispersion parameters in a wide range of their values in both aiding buoyancy and opposing buoyancy cases.

KEYWORDS: -Double Dispersion, Newtonian Fluid, Vertical Cone, Non-Darcy Porous Medium.

I. INTRODUCTION

Fluid-saturated with/without porous medium, by considering flow due to free convection, under different geometries has attracted attention of several researchers because of its necessities in scientific studies. To begin with, axi-symmetric shapes such as vertical and horizontal cylindrical, conical, and spherical shapes are utilized to recognize the flow behaviour due to convective mechanisms such as the viscous dissipation, external sources etc. Considering the field and flow dependency conditions on the fluid flow medium, various flow designs are being adopted. Different concepts describing the porous media and fluid-saturated porous media have appeared so far in a number of books, a number of monographs, and a number of reviews, to mention few (Vafai[1] - Nield and Bejan[3]). But the majority of literature explored only traditional viscous fluids passing through porous media with simple geometries.

A study on the fluid flows due to buoyancy forces over a cone-shaped geometry have been conducted by many authors as Alamgir [4], Pop and Takhar[5], Eco [6] and Awad et al. [7]. However, the convective flows along/over cylinders, flat/ stretching sheets, and non-flat surfaces have been undertaken by many investigators. In the ubiquitous of inertial effects, the Forchheimer model become unavoidable. At the pore scale level, the dispersion of thermal and solutal transports owing to the hydrodynamic incorporation turn out to be significant, as appeared in analysis of fluid flows over vertical plates (Cheng [8] - Srinivasacharya et al.[12]). These thermal and solutal dispersion developments have been mainly related to miscible displacement and solute spreading in porous media and are also important in many disciplines, to be specific, a few of them are ceramic processing, oil reservoir, heat storage beds, etc. In all the above said applications, there is use of cone-shaped bodies. However, there are only few articles are available in this regard. For instance, Kairi [13] considered double dispersion effects on buoyancy convective flow from a cone along vertical direction in a porous medium saturated with non-Newtonian liquids whereas RamReddy and Venkata Rao [14] analyzed the same effects over the truncated cone embedded in a non-Darcy porous medium saturated by nanofluids. In all the above studies, it is concluded that the natural convection driven by double dispersion engage in overall recreation of the transport of energy and concentration.

According to the author's knowledge, the effects of dual (i.e. thermal and solutal) dispersion on flow due to free convection over a vertical cone situated in a non-Darcy porous medium has not been discussed yet in the literature. Stimulated by the available results and the enormous promising industrialized needs, it is of dominant curiosity in this study to discuss the dispersion due to thermal and solutal transports over a cone in both opposing and aiding buoyancy cases.

II. DESCRIPTION OF MATHEMATICAL MODELLING

Consider an incompressible Newtonian fluid flow, due to buoyancy forces, over a heated vertical cone which is situated in a non-Darcy porous medium under steady state and laminar flow behaviours. Choose the two-dimensional geometry, as depicted schematically in Fig. (1). According to Eco [6], the local radius at a point located in momentum boundary layer and the radius of a cone can be approximated by $r = x \sin \Omega$. On the other hand, the flow equations of the present formulation are stringently valid with small peak slant cones

only. We assume the wall temperature is $T_w(x) = T_\infty + (T_w - T_\infty) \left(\frac{x}{L} \right)$ and wall concentration is

$C_w(x) = C_\infty + (C_w - C_\infty) \left(\frac{x}{L} \right)$, where L is a characteristic length. In addition, the fluid and the porous

medium are assumed to be in local thermodynamically equilibrium. The flow intensity is taken to be moderate and the permeability of the medium is presumed to be less in order to certify the applicability of the Forchheimer type flow model and negligence of boundary effect.

Using the approximation due to Boussinesq and the above-said assumptions, the basic equations governing for a viscous incompressible liquid, in the presence of thermal and solutal dispersion, are given by

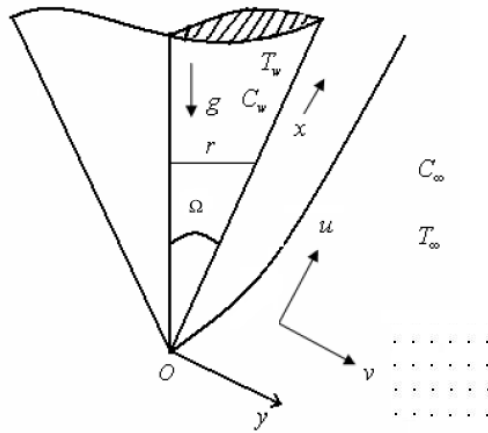


Figure 1: Physical model and coordinate system

$$\frac{\partial(ru)}{\partial x} + \frac{\partial(rv)}{\partial y} = 0,$$

(1)

$$\frac{1}{\varepsilon^2} \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) + \frac{vu}{K_p} + \frac{bu^2}{K_p} = \frac{v}{\varepsilon} \frac{\partial^2 u}{\partial y^2} + g \beta_T (T - T_\infty) \left(1 + \frac{\beta_C (C - C_\infty)}{\beta_T (T - T_\infty)} \right) \cos \Omega, \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \gamma d \left(\frac{\partial u}{\partial y} \frac{\partial T}{\partial y} + u \frac{\partial^2 T}{\partial y^2} \right),$$

(3)

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} + \xi d \left(\frac{\partial u}{\partial y} \frac{\partial C}{\partial y} + u \frac{\partial^2 C}{\partial y^2} \right). \quad (4)$$

where (u, v) are the Darcian velocities along x and y - axes respectively, $(T, C, g, \rho, \nu, \varepsilon, \beta_T, \beta_C)$ are the temperature, concentration, acceleration due to gravity, fluid density, kinematic viscosity, porosity, thermal and solutal expansion coefficients. Further, (d, α, D) are the pore diameter, thermal and molecular diffusivities, and (γ, ξ) are the thermal and solutal dispersions coefficients (See Ref.[11] & [12]).

The conditions on the wall and away from the wall are as follows

$$\text{On the wall, i.e., } y=0: u=v=0, T=T_\infty + (T_w - T_\infty) \left(\frac{x}{L} \right), C=C_\infty + (C_w - C_\infty) \left(\frac{x}{L} \right)$$

$$\text{Away from the wall, i.e. } y \rightarrow \infty: u \rightarrow 0, T \rightarrow T_\infty, C \rightarrow C_\infty. \quad (5)$$

Here the subscript w is used to denote the conditions at the wall whereas the outer edge conditions are represented by subscript ∞ .

Initially, the dimensionless set of quantities (x, y, r, u, v, T, C) are replaced by the following dimensionless counterparts:

$$X = \frac{x}{L}, Y = \frac{y}{L} Gr^{1/4}, R = \frac{r}{L}, U = \frac{u}{U_0}, V = \frac{v}{U_0} Gr^{1/4}, \bar{T} = \frac{T - T_\infty}{T_w - T_\infty}, \bar{C} = \frac{C - C_\infty}{C_w - C_\infty}.$$

Later, by utilizing the stream function Ψ by $U = \frac{1}{R} \left(\frac{\partial \Psi}{\partial Y} \right), V = -\frac{1}{R} \left(\frac{\partial \Psi}{\partial X} \right)$ and the transformations

$\Psi(X, Y) = R X f(Y), \bar{T}(X, Y) = X \theta(Y), \bar{C}(X, Y) = X \phi(Y)$ in the Eq. (1) to Eq. (4), we obtain the following system of nonlinear ordinary differential equations

$$\frac{1}{\varepsilon} f''' + \frac{1}{\varepsilon^2} (2ff'' - (f')^2) + \theta + B\phi - \frac{1}{Da \cdot Gr^{1/2}} f' - \frac{Fs}{Da} X \cdot f'^2 = 0, \quad (6)$$

$$\theta'' + Pr(DsX(f'\theta')' + 2f\theta' - f'\theta) = 0, \quad (7)$$

$$\phi'' + Sc(DcX(f'\phi')' + 2f\phi' - f'\phi) = 0. \quad (8)$$

The associated B.C. (5) in the form of f, θ, ϕ become

$$f = 0, f' = 0, \theta = 1, \phi = 1 \text{ at } Y = 0, \quad (9a)$$

$$f' \rightarrow 0, \theta \rightarrow 0, \phi \rightarrow 0 \text{ as } Y \rightarrow \infty. \quad (9b)$$

In the above equations, the prime denotes differentiation with respect to Y . In usual notations, $U_0 = [g\beta_T L(T_w - T_\infty) \cos \Omega]^{1/2}$ is the reference velocity, ε is the porosity, $Da = K_p / L^2$ is the Darcy parameter, $Fs = b/L$ is the Forchheimer parameter, $Gr = (U_0 L / \nu)^2$ is the global Grashof number, $Pr = \nu / \alpha$ is the Prandtl number, $Sc = \nu / D$ is the Schmidt number, $B = \beta_c (C_w - C_\infty) / [\beta_T (T_w - T_\infty)]$ is the buoyancy ratio, $Dc = \xi d Gr^{1/2} / L$ and $Ds = \gamma d Gr^{1/2} / L$ are respectively denote the solutal and thermal dispersion parameters. Note that $B < 0$ and $B > 0$ respectively represents the opposing and aiding buoyancies. Upon careful observation, it is reveals from Eqs. (6)-(8) that the concentration and energy equations will not give similarity profiles because of x -dependency. Thus, the numerical simulation may be conducted by treating $X = x/L$ as addition variable, for ease of analysis, to study the physical problem undertaken.

The non-dimensional skin friction $C_f = \frac{2\tau_w}{\rho U_0^2}$, Nusselt number $Nu = -\frac{L(k + k_d)}{kX(T_w - T_\infty)} \left[\frac{\partial T}{\partial y} \right]_{y=0}$

and the Sherwood number $Sh = -\frac{L(D + D_d)}{DX(C_w - C_\infty)} \left[\frac{\partial C}{\partial y} \right]_{y=0}$ are readily obtained in the form

$$C_f Gr^{1/4} = 2f''(0), Nu Gr^{-1/4} = -[1 + Pr X Ds f'(0)] \theta'(0),$$

$$Sh Gr^{-1/4} = -[1 + Sc X Dc f'(0)] \phi'(0).$$

where k is the molecular conductivity, k_d is the dispersion thermal conductivity, and D_d is the diffusivity of solutal dispersion.

III. SOLUTION AND DISCUSSION OF THE MATHEMATICAL MODELLING

The flow Eq. (6) accompanied with energy Eq. (7), concentration Eq. (8) and the B.Cs. (8), constitute highly complex non-linear non-homogeneous system of ODEs and closed-form solutions for these kinds of system cannot be obtained. Therefore, the Eqs. (6)-(8) together with the B.Cs.(8) are solved efficiently using the shooting method in conjunction with the Runge-Kutta technique of 4th order. Combination of these methods has been proved to be appropriate and gives precise outcomes for complex boundary layer equations. In this study, the maximum value of Y at ∞ (i.e. Y_∞) is indentified as $Y_\infty = 15$ at which the outer boundary conditions satisfy asymptotically. Further, the grid independent test has been conducted by taking 0.001, 0.01, and 0.05 as step sizes for ΔY . After careful examination and scrutiny, a step length of 0.001 is utilized in all cases for ΔY with the reasonable error less than 10^{-8} . Widespread mathematical simulations have been performed to explore the physical quantities for a wider choice of parameters.

In the limiting case, as $Ds = 0, Dc = 0, Da \rightarrow \infty, \varepsilon = 1$ and $B = 0$, the Eqs. (1) - (4) shrink to the equations of natural convective heat transport over a cone (Eco [6]). When $Da \rightarrow \infty, Fs = 0$ and $\varepsilon = 1$, the Eqs. (1)-(4) coincides with the equations of free convection mass and heat transfer about a cone [15]. The comprehensive comparison with outcomes provided by Eco [6] and RamReddy [15] as shown in the Table (1) and found to be in perfect agreement.

Table (1): Similarity of results for Newtonian fluid about a cone with $Ds = Dc = 0, Da \rightarrow \infty, \varepsilon = 1, B = 0$

Pr	$f''(0)$			$-\theta'(0)$		
	Eco [6]	RamReddy [15]	Present results	Eco [6]	RamReddy [15]	Present results
1.0	0.681482	0.681482	0.681445	0.638859	0.638859	0.638812
10.0	0.433269	0.433272	0.433145	1.275548	1.275554	1.275192

To have a sympathetic findings of the Eqs, (1)-(4) with B.C.(5), results for the flow of fluid, heat and mass transfers are evaluated, in $B > 0$ and $B < 0$ cases, for wide range of thermal dispersion parameter (Ds), Darcy parameter(Da), Forchheimer parameter(Fs) and solutal dispersion parameter(Dc).

Case(a): Newtonian fluid flow due to buoyancy over a cone situated in a Darcy porous medium - Effects of thermal and solutal dispersion parameters

The present mathematical modelling represents the natural convective flow of Newtonian fluid over a vertical cone situated in a Darcy porous medium with double dispersion when inertial effects are absent (i.e. $Fs \rightarrow 0$). In the Figs. 2(a) to 2(c), for $Dc = 0.3, Gr = 10, \varepsilon = 0.5, Pr = 1.0, Sc = 1.0$ and $X = 0.5$, the stream-wise variations of rates of heat and mass transfers, skin friction coefficient at the surface of the cone for wider range of Ds in both cases of aiding buoyancy ($B > 0$) and opposing buoyancy ($B < 0$). All these physical quantities $C_f Gr^{1/4}, Nu Gr^{-1/4}, Sh Gr^{-1/4}$ are found to be behave similarly and are enhancing nonlinearly in the stream-wise direction when Ds enhances in both the cases $B < 0$ and $B > 0$. As $Ds \rightarrow 0$ and $Dc \rightarrow 0$, the problem of porous medium saturated by a Newtonian fluid without double dispersion effects is noticed. The coefficient of skin friction, rates of mass and heat transfers are less in the nonexistence of Ds when compared to the existence of Ds . When we take Ds into consideration, conduction over convection favours by the energy equation. Subsequently, the dominance of more thermal conduction is seen with an addition of Ds to the energy equation. An additional significant reality established by Figs. 2(a) to 2(c) is that lower values of the skin friction, mass and heat transfer rates are produced in opposing buoyancy case than those of aiding buoyancy. In general, this kind of fluid-saturated porous media with improved properties of heat dispersion may possibly produce improved characteristics of heat transfer that possibly will be mandatory in numerous built-up applications, specifically packed bed reactors, nuclear waste disposal, etc.

The nonlinear growth of these three physical quantities (viz. $C_f Gr^{1/4}, Nu Gr^{-1/4}, Sh Gr^{-1/4}$) are seen with $B > 0$ whereas the coefficient of skin friction and rate of heat transfer are found to fall down nonlinearly but the rate of mass transfer is seen to grow nonlinearly with Dc when $B < 0$, as illustrated in Fig.(3), for fixed values of other parameters. Like thermal dispersion parameter, higher values of $C_f Gr^{1/4}$,

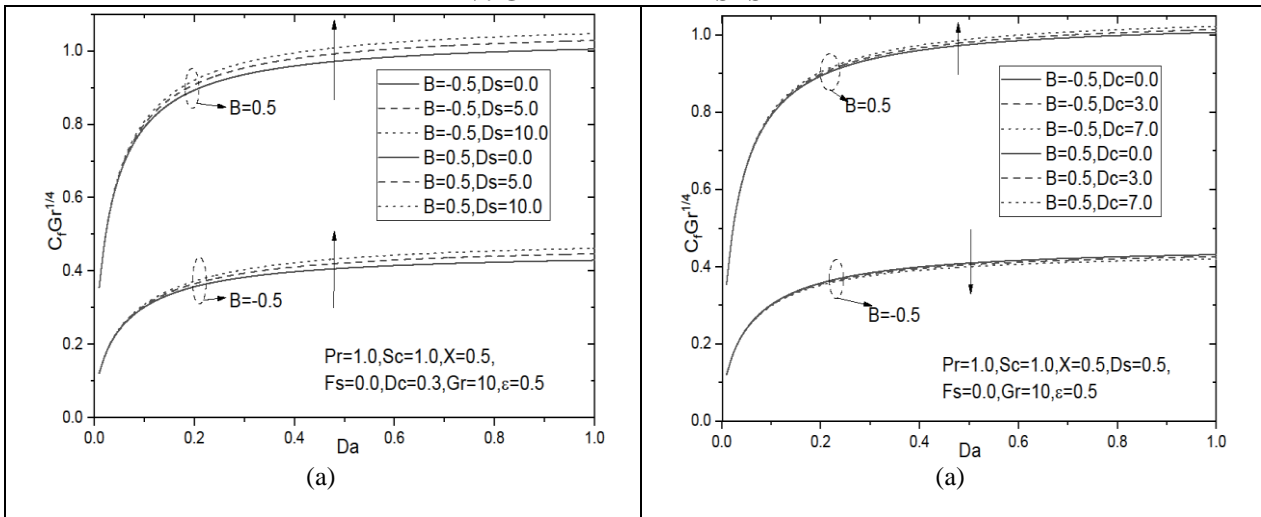
$NuGr^{-1/4}$, $ShGr^{-1/4}$ are noticed in the case of aiding buoyancy in comparison to that of opposing buoyancy case. Further, it is clear from Figs. 2(a) to 3(c) that the coefficient of skin friction, and rates of heat and mass transfers are increasing nonlinearly with the increasing values of Darcy parameter Da. It is also noticed from this analysis that the values of $C_fGr^{1/4}$, $NuGr^{-1/4}$, $ShGr^{-1/4}$ are less in the Newtonian fluids without porous medium when compared to Newtonian fluids saturated porous medium.

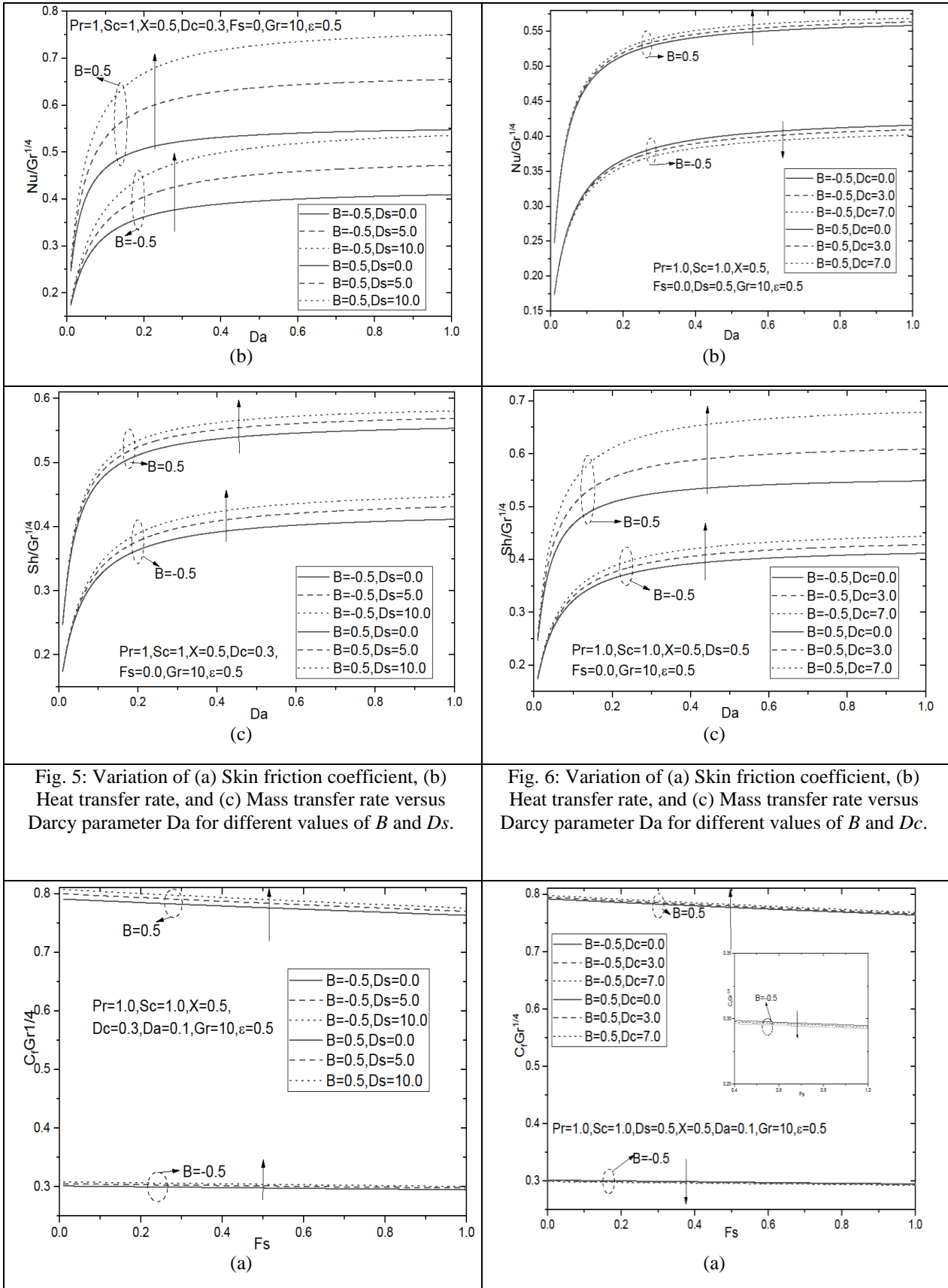
Case(b): Newtonian fluid flow due to buoyancy over a cone situated in a non-Darcy porous medium - Effects of thermal and solutal dispersion parameters

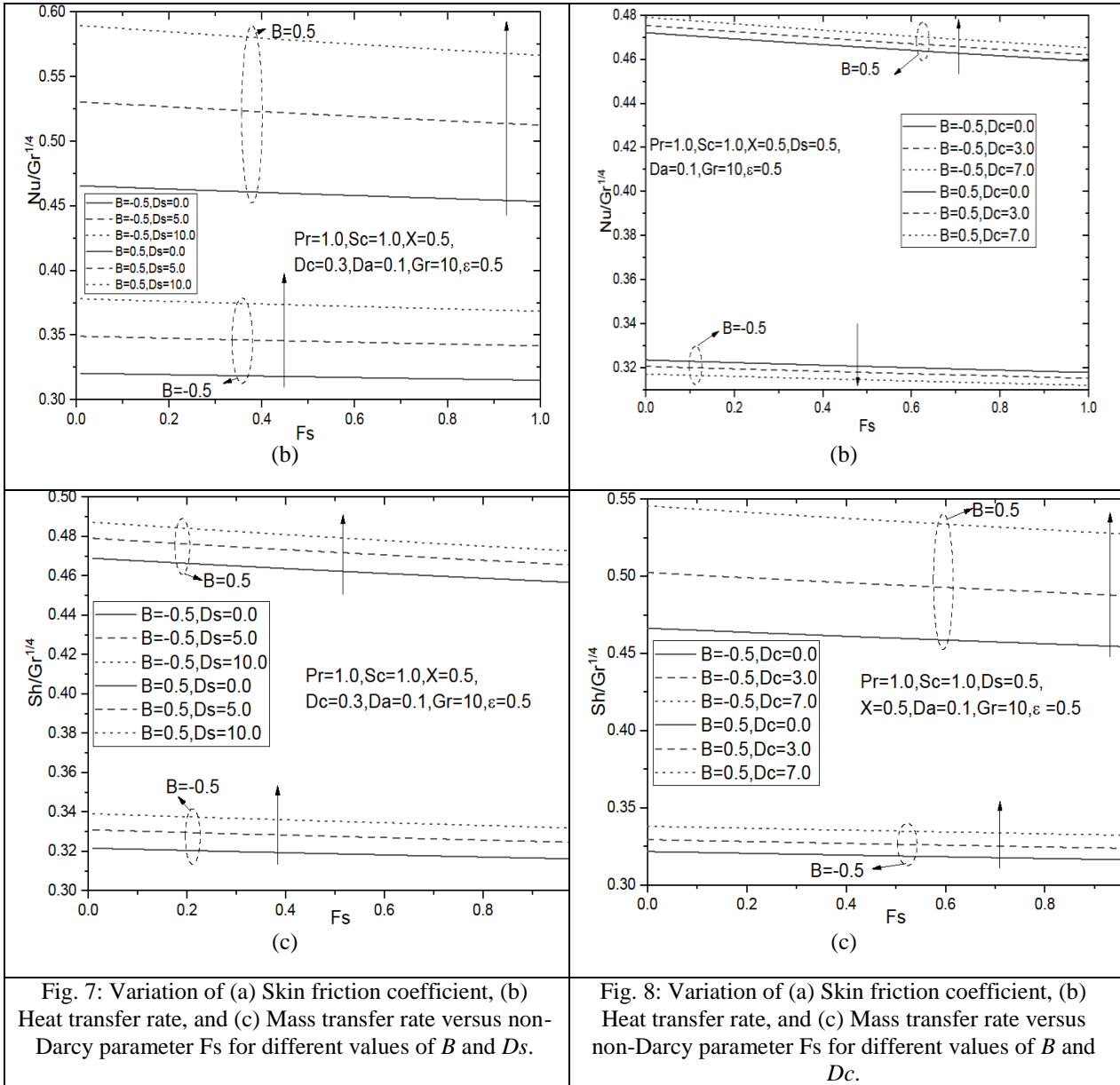
In this case, the free convective transport over a vertical cone situated in a non-Darcy porous medium saturated by a Newtonian fluid is considered and effects of different physical parameters on various physical quantities are plotted in Figs.4(a) to 5(c) in both aiding and opposing buoyancy cases. It is observed from Figs. 4(a) to 4(c) that the skin friction, heat and mass transfer coefficients enhanced linearly in both the cases of aiding and opposing buoyancy for enhancing values of the thermal dispersion parameter D_s . But, with the enhancement of solutal dispersion parameter D_c , the growth in the values of $C_fGr^{1/4}$, $NuGr^{-1/4}$, $ShGr^{-1/4}$ are noticed in the aiding buoyancy case whereas the reduction in the values of $C_fGr^{1/4}$ and $NuGr^{-1/4}$ and growth in the value of $ShGr^{-1/4}$ are observed in the opposing buoyancy case, as depicted in Figs. 5(a) to 5(c). Like the thermal dispersion parameter D_s , higher values of the skin friction, heat and mass transfer rates are noticed in the case of aiding buoyancy in comparison to those of opposing buoyancy case. The similar behavior is found in the case of Darcy porous medium, which are shown above.

If the Forchheimer parameter is enhanced in the medium, then the resistance of the fluid flow will become increasingly more and subsequently increase temperature values. As the fluid is decelerated with an increase of non-Darcy parameter, hence an energy is dissipated as heat and serves to decrease the heat and mass transfer coefficients within the boundary layers. Unlike Darcy parameter Da, it is

IV. GRAPH DATABASES







found from Figs. 4(a) to 5(c) that the values of $C_f Gr^{-1/4}$, $Nu Gr^{-1/4}$, $Sh Gr^{-1/4}$ are decreasing linearly with the increasing values of non-Darcy parameter (F_s). Since the values of $C_f Gr^{-1/4}$, $Nu Gr^{-1/4}$, $Sh Gr^{-1/4}$ are less in the non-Darcy porous medium saturated by a Newtonian fluid comparing to the Darcy porous medium, which may be beneficial in flow, temperature and concentration field control of polymer processing. Hence the non-Darcy parameter has an important role in controlling the flow field.

V. CONCLUSION

In this paper, the effects thermal and solutal dispersions on natural convective flow of Newtonian fluid over the cone with a fixed apex half angle, pointing vertically downwards embedded in a non-Darcian porous medium, are investigated. Numerical investigation of the non-dimensional flow governing equations is established using the 4th order RK scheme in conjunction with the shooting technique and also provided the comparative table to show the accuracy of present investigation. The results are provided by the comparative study between fluid saturated Darcy and non-Darcy porous medium. The major conclusion, obtained in this analysis, are as follows:

(1) Nonlinear increments of the skin friction coefficient, rates of heat and mass transfer are observed with an increase of thermal dispersion in the case of Darcy porous medium. But, the linear increments on these physical quantities are seen in the non-Darcy porous medium. However, these results are true in both cases of aiding and opposing buoyancy.

(2) The heat transfer rate and skin friction coefficient decrease nonlinearly while the rate of mass concentration transfer increases nonlinearly with an increase of dispersion of solutal transport in the Darcy porous medium with opposing buoyancy. But the mass transfer rate, skin friction coefficient, heat transfer rate increases nonlinearly in Darcy porous medium with an increase of solutal dispersion in the case of aiding buoyancy. But, the respective linear increments/decrements on these physical quantities are seen in the non-Darcy porous medium.

(3) In both aiding and opposing buoyancy cases: the skin friction, heat and mass transfer rates are found to be more and increases nonlinearly in the fluid-saturated porous medium but these physical quantities decrease linearly in the non-Darcy porous medium.

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